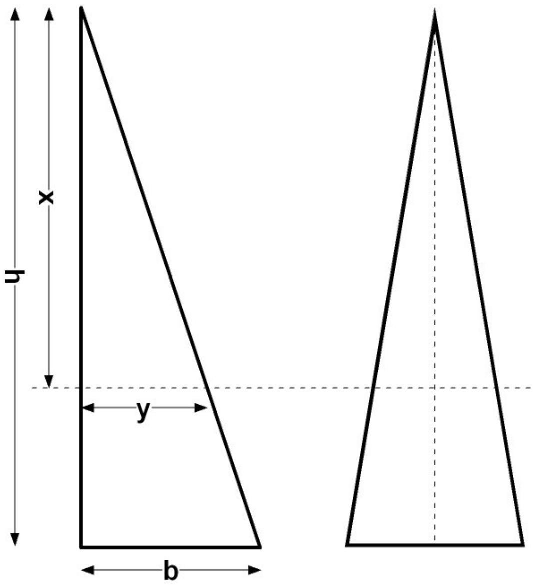
Finding the vertical area bisector (approximate height of center of effort for triangular sails)



From diagram:

$$A = \frac{1}{2}\cdot b\cdot h \qquad A_x = \frac{1}{2}\cdot y\cdot x \qquad \frac{h}{b} = \frac{x}{y} \qquad y = \frac{b}{h}\cdot x$$

Set "x" as center of effort height:

$$A_x = \frac{1}{2}\cdot A$$

Substitute and simplify:

$$\frac{1}{2}\cdot y\cdot x = \frac{1}{2}\cdot\frac{1}{2}\cdot b\cdot h \qquad \frac{b}{h}\cdot x^2 = \frac{1}{2}\cdot b\cdot h \qquad x^2 = \frac{h^2}{2} \qquad x = \frac{h}{\sqrt{2}}$$

Find height from base:

$$H_{ce} = h - x \qquad H_{ce} = h - \frac{h}{\sqrt{2}} \qquad H_{ce} = h\cdot\left(1 - \frac{1}{\sqrt{2}}\right) \qquad H_{ce} = D\cdot h \qquad D = 0.293$$

Scaling sail area for varying wind speed targets:
Sail forces are proportional to sail area and wind speed squared:
(C is a constant that depends on air density and sail Lift/drag properties)

$$F = C\cdot A\cdot V^2$$

Heeling moment is proportional to sail forces and center of effort height:

$$M = F\cdot H_{ce} \qquad M = C\cdot A\cdot V^2\cdot H_{ce}$$

To maintain a constant heeling moment for different wind speeds V and V':
(Assuming the two sails are similar in shape and aspect ratio)

$M = M'$ $\qquad$ $C \cdot A \cdot V^2 \cdot H_{ce} = C \cdot A' \cdot V'^2 \cdot H'_{ce}$

simplifies to:

$$A \cdot V^2 \cdot H_{ce} = A' \cdot V'^2 \cdot H'_{ce}$$

Solve for scaling factor as function of wind speed change ("s" is linear scaling factor):

$A = \frac{1}{2} \cdot b \cdot h$ $\qquad$ $H_{ce} = D \cdot h$ $\qquad$ $A' = \frac{1}{2} \cdot b' \cdot h'$ $\qquad$ $H'_{ce} = D \cdot h'$ $\qquad$ $h' = s \cdot h$ $\qquad$ $b' = s \cdot b$

Find "s":

$\frac{1}{2} \cdot b \cdot h \cdot V^2 \cdot (D \cdot h) = \frac{1}{2} \cdot b' \cdot h' \cdot V'^2 \cdot (D \cdot h')$ $\qquad$ $b \cdot h^2 \cdot V^2 = b' \cdot h'^2 \cdot V'^2$

$b \cdot h^2 \cdot V^2 = s \cdot b \cdot (s \cdot h)^2 \cdot V'^2$ $\qquad$ $V^2 = s^3 \cdot V'^2$

$$s = \left(\frac{V'}{V}\right)^{\frac{-2}{3}}$$

Compute and plot results:

$\frac{V'}{V} = R$ $\qquad$ $s(R) := R^{\frac{-2}{3}}$ $\qquad$ $R := 0.1, 0.2 .. 3$