Finding the vertical area bisector (approximate height of center of effort for triangular sails)

From diagram:

\[ A = \frac{1}{2}bh \quad A_x = \frac{1}{2}yx \quad \frac{h}{b} = \frac{x}{y} \quad y = \frac{b}{h}x \]

Set "x" as center of effort height:

\[ A_x = \frac{1}{2}A \]

Substitute and simplify:

\[ \frac{1}{2}y \cdot x = \frac{1}{2} \cdot \frac{1}{2}b \cdot h \quad \frac{b}{h} \cdot x = \frac{1}{2} \cdot b \cdot h \quad x^2 = \frac{h^2}{2} \quad x = \frac{h}{\sqrt{2}} \]

Find height from base:

\[ H_{ce} = h - x \quad H_{ce} = h - \frac{h}{\sqrt{2}} \quad H_{ce} = h \left( 1 - \frac{1}{\sqrt{2}} \right) \quad H_{ce} = D \cdot h \quad D = 0.29^2 \]

Scaling sail area for varying wind speed targets:
Sail forces are proportional to sail area and wind speed squared:
(C is a constant that depends on air density and sail Lift/drag properties)

\[ F = C \cdot A \cdot V^2 \]

Heeling moment is proportional to sail forces and center of effort height:

\[ M = F \cdot H_{ce} \quad M = C \cdot A \cdot V^2 \cdot H_{ce} \]
To maintain a constant heeling moment for different wind speeds $V$ and $V'$:
(Assuming the two sails are similar in shape and aspect ratio)

$$M = M' \quad C \cdot A \cdot V^2 \cdot H_{ce} = C \cdot A' \cdot V'^2 \cdot H'_{ce}$$

simplifies to:

$$A \cdot V^2 \cdot H_{ce} = A' \cdot V'^2 \cdot H'_{ce}$$

Solve for scaling factor as function of wind speed change ("s" is linear scaling factor):

$$A = \frac{1}{2} b \cdot h \quad H_{ce} = D \cdot h \quad A' = \frac{1}{2} b' \cdot h' \quad H'_{ce} = D \cdot h' \quad h' = s \cdot h \quad b' = s \cdot b$$

Find "s":

$$\frac{1}{2} b \cdot h \cdot V^2 \cdot (D \cdot h) = \frac{1}{2} b' \cdot h' \cdot V'^2 \cdot (D \cdot h') \quad b \cdot h \cdot V^2 = b' \cdot h' \cdot V'^2$$

$$b \cdot h \cdot V^2 = s \cdot (s \cdot h) \cdot V^2 \quad V^2 = s^3 \cdot V'^2$$

$$s = \left( \frac{V'}{V} \right)^{-2}$$

Compute and plot results:

$$\frac{V'}{V} = R \quad s(R) = R^{-\frac{2}{3}} \quad R := 0.1, 0.2..3$$